

# Channel Width Assignment using Relative Backlog: Extending Back-pressure to Physical Layer

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## ABSTRACT

With recent advances in Software-defined Radios (SDRs), it has indeed become feasible to dynamically adapt the channel widths at smaller time scales. Even though the advantages of varying channel width (e.g. higher link throughput with higher width) have been explored before, as with most of the physical layer settings (rate, transmission power etc.), naively configuring channel widths of links can in fact have negative impact on wireless network performance. In this paper, we design a cross-layer channel width assignment scheme that adapts the width according to the backlog of link-layer queues. We leverage the benefits of varying channel widths while adhering to the invariants of back-pressure utility maximization framework. The presented scheme not only guarantees improved throughput and network utilization but also ensures bounded buffer occupancy and fairness.

## Categories and Subject Descriptors

C.2.1 [Computer-Communication Networks]: Network Architecture and Design

## General Terms

Design, Algorithm, Performance

## Keywords

Utility maximization, Back-pressure, Channel width adaptation, IEEE 802.11

## 1. INTRODUCTION

Even though the advantages of variable channel width assignment have been explored before [2], a few fundamental questions surrounding the problem have largely remained unanswered. Some of the questions are -

(1) Do link-level benefits of higher throughput using increased channel width come at other network-wide performance penalties?

(2) How can we assign channel width to links such that flows can achieve maximum throughput while maintaining fairness among the flows?

(3) Since variable channel width can introduce significant variations among achievable per-link throughput, how can we design a channel width assignment scheme that can guarantee stability of queues and bounded buffer occupancy?

This paper attempts to answer the questions by utilizing back-pressure principles of utility maximization framework

for the purpose of variable channel width assignment. We show that substantial gains in throughput and network utilization can be achieved along with improved fairness when back-pressure framework is used as a directive for channel width assignment. The central idea is to assign higher channel widths to queues with larger backlog (relative to its interference neighborhood).

## 1.1 The Back-pressure Framework

We assume a network in which transmission time is divided into slots. For a network graph  $G = (V, E)$ , let  $f \in F$  denote a flow from  $s(f)$  to  $d(f)$ . Let  $l(u, v) \in E$  denote a link between node  $u$  and  $v$ , and  $\gamma_{l(u, v)}$  be the transmission rate of link  $l(u, v)$ . Transmission rates of all links in  $E$  are presented by  $\Gamma = \{\gamma_{l(u, v)}, l(u, v) \in E\}$ . Let  $\chi$  be the set of all possible combinations of rates at which links can operate. Every node  $u \in V$  maintains a separate queue (Per Destination Queue - PDQ) for destinations of all flows in  $F$ . Packets received by  $u$  for a flow  $f$  destined to  $d(f)$  are queued in  $Q_u^{d(f)}$ . Let  $|Q_u^{d(f)}(t)|$  denote the size of the PDQ maintained at node  $u$  for destination  $d(f)$  at time  $t$ . Every node shares its PDQ length information with all its neighbors at the beginning of time slot  $t$ . Using this, a node  $u$  calculates  $D_{l(u, v)}^{d(f)}(t) = |Q_u^{d(f)}(t)| - |Q_v^{d(f)}(t)|$  for all  $l(u, v) \in E$  and all  $f \in F$ . Now, for every link  $l(u, v) \in E$ , let  $\Delta_{l(u, v)}(t) = \max_{f \in F} (D_{l(u, v)}^{d(f)}(t))$ . Back-pressure scheduling [3] suggests that  $\Gamma$  at time  $t$  should be chosen such that -

$$\Gamma(t) = \max_{\Gamma \in \chi} \sum_{l(u, v) \in E} (\gamma_{l(u, v)} \Delta_{l(u, v)}(t)) \quad (1)$$

It was proved in [3] that a routing/scheduling policy that can achieve a solution of Equ.1 is throughput optimal. Unfortunately, the above mentioned problem is proven to be NP-hard in wireless case due to interference constraints.

## 2. CHANNEL WIDTH ASSIGNMENT

It can be observed that any approximation of solution to Equ.1 should allow PDQs with higher backlog to transmit at a proportionally higher transmission rates. This higher transmission rate can be achieved by assigning the endpoints of PDQs (radio interfaces) wider channel widths.

Let  $P(t)$  be the set of all PDQs in the network at time  $t$ . For any PDQ  $p_i$  with backlog  $b(p_i)$ , let  $N(p_i)$  be the set of links whose PDQs interfere with the link of  $p_i$ . Let  $\max(p_i)$  (and  $\min(p_i)$ ) be the backlog of PDQ with maximum (minimum) backlog in  $N(p_i)$ . Algorithm 1 presents two *centralized* and *greedy* schemes of channel width assignment. The first scheme (Algo. 1 - Case 1) targets a *continu-*

ous assignment in which a PDQ can be assigned any width (in Hz) from total spectrum width of  $D$  Hz (60 MHz here as in most 802.11 radios). It determines the relative backlog of a PDQ as compared to its neighborhood PDQs in order to proportionally assign a channel width to it. In the case of Algo. 1 - Case 2, only a few pre-selected channel widths can be used by each PDQs. As in 802.11 standards, we choose 4 discrete levels of 5, 10, 20 and 40 MHz.

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**Algorithm 1** Channel Width Allocation

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**CASE 1: CONTINUOUS**

At the beginning of each time slot  $t$   
Sort  $P(t)$  in descending order of backlog  
**for**  $i = 1 \rightarrow |P(t)|$  **do**  
channel width  $cw(p_i) = \frac{(b(p_i) - \min(p_i)) \cdot D}{\max(p_i) - \min(p_i)}$   
**if**  $cw(p_i)$  width not available in  $N(p_i)$  **then**  
assign maximum available width in  $N(p_i)$  to  $p_i$   
**end if**  
**end for**

**CASE 2: DISCRETE**

$widthArray[4] = \{5, 10, 20, 40\}$   
At the beginning of each time slot  $t$   
Sort  $P(t)$  in descending order of backlog  
**for**  $i = 1 \rightarrow |P(t)|$  **do**  
 $l = \frac{(b(p_i) - \min(p_i)) \cdot 4}{\max(p_i) - \min(p_i)}$   
 $cw(p_i) = widthArray[l]$   
**while**  $cw(p_i)$  width not available in  $N(p_i)$  **do**  
 $l = l - 1; cw(p_i) = widthArray[l]$   
**end while**  
**end for**

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The scheme also reduces the transmission power of radios operating at a lower channel width in such a way that communication range of every node remains constant. Also, note that the number of radios necessary in order for both channel width allocation strategies to work can be very high (e.g. in worst case,  $D$  radios of continuous and  $D/5$  radios discrete schemes). This along with centralized nature of scheme makes it difficult to implement in practice. Nevertheless, the scheme works well as a proof-of-concept and sets important performance benchmarks for our ongoing distributed protocol design.

### 3. SIMULATION RESULTS

We choose a  $7 \times 7$  grid with 10 randomly chosen source-destination pairs. Each source node generates 1500 bytes packets using poisson random process with mean of 1350 packets/second. Bi-directional protocol interference model is used for determining the link interference relationships. Also, utility based source node injection rate control [1] is employed. The results of simulation and observations are shown in Figs. 1 and 2. We compare the schemes with two other schemes. First, a *fixed width channel assignment* is used which utilizes back-pressure scheduling but 60 MHz spectrum is divided into 3 channels of 20 MHz fixed widths. Second, a *random scheme* in which the same 3 channels are used but instead of scheduling the links using back-pressure policy, links are randomly chosen for scheduling.

### 4. DISTRIBUTED PROTOCOL DESIGN

We are currently developing a back-pressure based distributed channel width assignment protocol on CSMA/CA

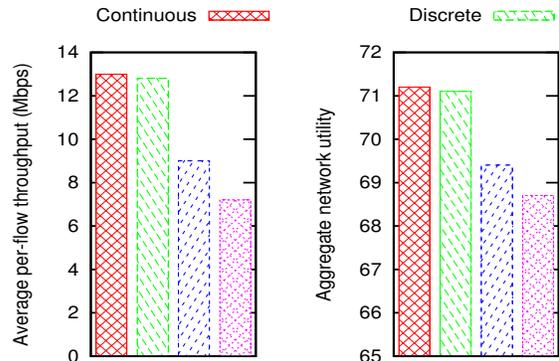


Figure 1: (a) Average per-flow throughput ( $x_f$ ) increases significantly (by factor of 1.4) in variable channel width case, (b) Network Utility ( $\sum_{f \in F} \log(x_f)$ ) also increases

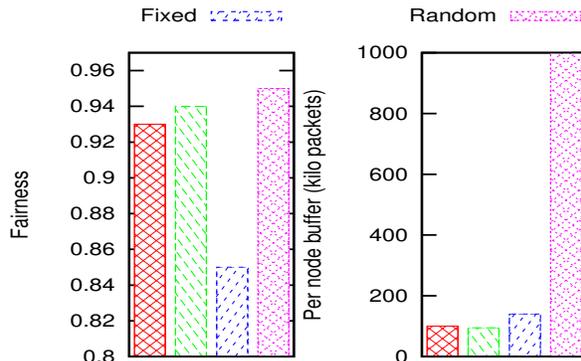


Figure 2: (a) Jain's fairness index ( $\frac{(\sum_{i=1}^{|F|} x_i)^2}{\sum_{i=1}^{|F|} x_i^2}$ ) increases because backlogged queues are served at a faster rate in variable channel width cases, (b) Note that buffer occupancy reduces by 25% as compared to random scheme. Even with comparison to fixed-width case, buffer occupancy reduces while increased fairness, throughput and utility in case of varying channel widths

MAC that can estimate the benchmark set by centralized scheme presented here. Apart from distributed selection of channel mid-point and width, and transmission synchronization, the problem becomes further more challenging due to the fact that in real-world scenarios each node possesses only a few limited number of radios.

### 5. ACKNOWLEDGMENTS

This work is supported by the U.S. Army Research Office (ARO) under grant W911NF-08-1-0105 managed by NCSU Secure Open Systems Initiative (SOSI). The contents of this paper do not necessarily reflect the position or the policies of the U.S. Government.

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